Comment on the paper 'A cosmological limit on the possible variation of G'

V. Canuto★ **and S.-H. Hsieh**† *NASA, Goddard Institute for Space Studies, Goddard Space Flight Center, New York, N.Y. 10025, USA*

Ever since Dirac (1937) proposed a possible variation of the gravitational constant G with cosmological time, there have been various attempts to show that such a variation can be ruled out from known astrophysical or cosmological observations. These conclusions have been based on analyses using otherwise standard dynamical equations aside from the substitution of G by a function of time. Recently, Barrow (1978) argued that the observed light element abundance gives a much more stringent limit on the temporal evolution of G and therefore imposes a severe constraint on proposed theories of gravity in which G is allowed to evolve. In this note, we wish to point out that Barrow's claim involves two serious conceptual errors, which invalidate his analysis.

The first one consists in assuming that in general the implications of a varying G can be fully accounted for by simply changing G into $G(t)$ in the standard Einstein equations while keeping the law of conservation of energy unaltered. This is, however, not correct since if G varies, the new conservation law is

$$(GT^{\mu\nu})_{;\nu=0}, \tag{1}$$

yielding for matter and radiation,

$$\rho_m \sim \frac{1}{G} \frac{1}{R^3}, \qquad \rho_\gamma \sim \frac{1}{G} \frac{1}{R^4}. \tag{2}$$

This implies that Einstein equations only tell us about the product $G\rho$ and not about ρ and G separately. Since the rate of expansion of the Universe during the radiation dominated era is proportional to $(G\rho_\gamma)^{-1/2}$, it is incorrect to conclude that a varying G would have accelerated the expansion rate to the point of leaving no time for nucleosynthesis to occur.

With regard to the scale covariant theory of gravitation, it has been shown by Canuto & Hsieh (1979) that within that framework a variable G necessarily implies modified conservation laws which in turn lead to modified thermodynamic relations.

In particular, it was demonstrated that the radiation energy density ρ_γ is given by

$$\rho_\gamma \sim \frac{1}{\beta^2 G} \frac{1}{R^4} \sim \frac{\beta^2}{G} T^4. \tag{3}$$

★ Also with Physics Department, CCNY, N.Y.
† NAS–NRC Research Associate.

The relation $\rho_\gamma \sim T^4$ used by Barrow is inconsistent with equation (3), thus invalidating the applicability of his conclusion to our theory.

The second conceptual error lies in the assumption that the different theories of varying G, proposed so far, differ only in the way G depends on the cosmological time (see Barrow's remark preceding his equation 5).

In fact, based on different fundamental assumptions, these different theories yield different dynamical equations and different conservation laws.

Barrow's analysis that allows a general power law dependence of G on time, but keeps standard conservation laws, does not represent *any* of the proposed theories he cited.

References

Barrow, J. D., 1978. *Mon. Not. R. astr. Soc.,* **184**, 677.
Canuto, V. & Hsieh, S.-H., 1979. *Astrophys. J. Suppl.,* **41**, 243, 263, 301.
Dirac, P. A. M., 1937, *Nature,* **139**, 323.